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# Optimisation of Bids for a Public Auction Involving the Delivery of an Infrastructure Project Using a Package of Numerical Procedures

**Sławomir Juszczyk, Jerzy Tymiński***

**Abstract:** The article deals with the preparation of bids for public auctions organised for the purposes of a public procurement system, which are distinguished by their financial effectiveness. It discusses the requirements for entering into a tendering process, giving special attention to the analysis of the potential structure of costs that constitute a major element of the process. An example of the application of IT tools developed with the open source platform for numerical computation Scilab is provided. The illustrative calculations concern two firms bidding at a public auction for a road infrastructure project. The bids are evaluated and based on the proposed project completion time. The example shows the possibilities of analysing project profitability with respect to variations in market conditions such as project costs, the discount rate and the predicted increase in output.

**Keywords:** investment projects, a tender, bid optimisation, numerical procedures, Scilab software package

## Introduction

Public auctions organised within a public procurement system are one of the major types of tendering. An important characteristic of this form of negotiations is that public auctions are economically effective for both the awarding party and the bidders. The selection of the successful contractor is preceded by an analysis of the potential participants that must be willing and capable of delivering the object of the tender.

The article discusses the application of IT tools in support of decision-making processes related to participation in an auction. The calculations that have been made take account of different aspects of uncertainty. Two hypothetical firms are considered which tender at a public auction for a long-term strategic project involving the construction of a piece of infrastructure. The bids are evaluated and based on two criteria: the project completion date and the bidder's capacity. The article explains how contract profitability can be assessed allowing for variations in market factors such as costs, the discount rate and demand. Because this type of project has a strategic character, an increase in bidders' production capacity, particularly in their profits, is also considered. Addressing these factors allows

* dr hab. Sławomir Juszczyk, prof. SGGW, dr Jerzy Tymiński, prof. WSGK.

a more in-depth analysis of learning processes to be conducted in the context of decision-making (Łyszkiewicz 2000, p. 199).The computer simulation used for illustrative purposes was performed with the Scilab software package (the open source platform for numerical computation).

In a market economy, global flows of capital significantly stimulate the growth of countries that cooperate and trade with one another. The free flow of modern technologies that usually involve major capital expenditures brings up the question about how effective these undertakings are. In tendering procedures, the economic effectiveness of undertakings that are innovative while offering considerable production capacity should be determined already at the stage of negotiations.

The tendering system used to procure construction work (regulated by the act – Public Procurement Law) operates on the single market rule which requires all Community-based firms to be given equal access to public procurement tenders. This solution, while allowing firms to participate in tendering procedures announced all over the European Union, makes negotiations prone to risk and uncertainty (Tymiński 2003).

To decide which bid is the most appropriate considering the evaluation criteria, the awarding party may choose one of several approaches (Jaworski 1999). The contract may be granted through:

– an open tendering procedure where all submitted bids are evaluated against multiple criteria,
– a restricted tendering procedure where only one bidder's proposal is evaluated,
– a tendering procedure leading to competitive negotiations.
– single-source procurement (this approach is adopted when a tendering procedure does not seem to be a practical solution),
– request for quotations that the awarding entity sends to the contractors it has chosen. In this case, a contractor must fulfil only one criterion (the price) to be awarded the contract.

A special situation is public auctions where the participants bid publicly for an object that has some objective value that they do not know (Łyszkiewicz p. 325).

The type of tendering procedure determines how complex tools offered by theory and practice will be used to make evaluations and decisions. Widely used are econometric models, and recently also neural networks. Econometric modelling allows the best bid value to be estimated, as well as analysing the possible impacts of factors such as price, warranty period, etc., on its level. However, when uncertainty needs to be addressed, specialist decision-making and evaluation tools can be employed, e.g. game theory, mass service theory, reliability theory or the theory of entropy.

In the evaluation of infrastructure projects that take a long time to be constructed more factors affecting the production capacity of firms participating in tendering procedures and winning contracts can be analysed. In addition to the transfers of modern technologies the factors include also learning-by-doing processes.

## 1. Arguments for Learning-by-doing processes to be used in the evaluation of the future production capacity of firms participating in a public auction

The effects of learning by doing appear because of production capacity expanding in time. A steadily growing cumulative volume of output reduces the unit cost and thereby increases the profit per unit. In a competitive environment, this may cause prices to fall and demand to grow. The mechanism of this process is illustrated in Figure 1.

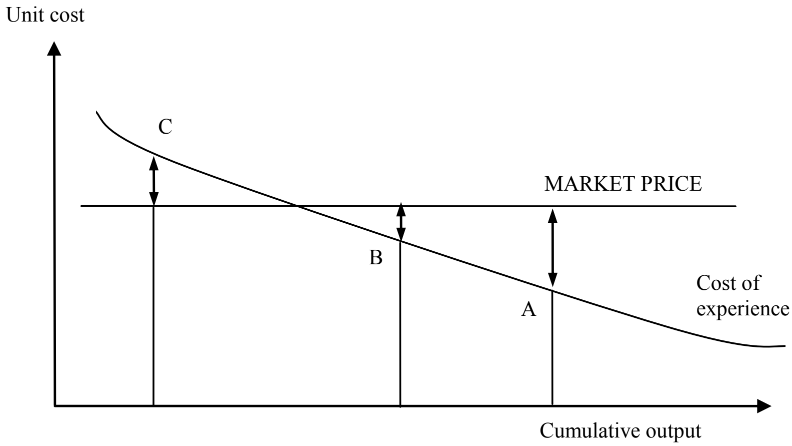


**Figure 1.** Learning-by-doing processes in relation to a firm's competitive position

Source: Ferens (2001).

The above chart presents a learning curve for three rival firms: A, B, C. Their competitive positions are explained through a decline in average production costs. The effects of learning processes can be gained owing to:

- unit costs decreasing as the production capacity and sales expand,
- economies of scale, the main source of which is falling unit fixed costs (because of output increasing faster than fixed costs, etc.),
- improving workers' skills, which translates into higher productivity, proficiency, better organisation of work, etc.,
- innovativeness, mainly due to the use of modern technologies,
- substitution of capital for labour,
- other optimisation factors, particularly within the supply chain.

## 2. The nature and the process of tendering at a public auction

Auctions where the participants, including international corporations, present their maximum bids that are basically determined with respect to their production costs constitute an important segment of public procurement. These auctions are usually won by organisations that in addition to having the necessary production capacity and the direction of strategic development consistent with the purpose of the auction (e.g. emphasising investment or learning) is also the least expensive and the most effective of all bidders. In practice this type of action is called an English auction[1]and its participants make public bids in turn. The successful bidder is the one that undertakes to complete the project in the shortest time possible. Hence $t_j \in (0, T)$, where $T$ – time needed to carry out the tendered project. This condition allows an optimal decision-making criterion expressed by the maximization of profit for a given discount rate β. The return must be established *a priori* taking into account a possible growth in output and an extrapolated amount of the project execution costs.

The first to make a bid is firm A that undertakes to complete the tendered project in time ($t_A$) and then firm B submits its offer ($t_B$). If A's bid is better than B's then it wins the auction and starts to carry out the project.

However, if firm B outbids firm A by offering a shorter time $t_i$ than it is granted the project. Assuming a continued capitalisation of interest, the return of the successful bidder winning the auction with time $t_i$ will be given by the formula:

$$Z_{akt} = \int_0^{t^*} (P_A(t) + \vartheta(t) \cdot C(P_A(t) + P_B(t)) \cdot e^{-\beta t} dt + \int_{t^*}^{T} (P_A(t) \cdot C(P_A(t)) \cdot e^{-\beta t} dt + \\ - \int_0^{t^*} K_A(P_A(t) + \vartheta(t)\gamma(t)) \cdot e^{-\beta t} dt - \int_{t^*}^{T} K_A(P_A(t)) \cdot e^{-\beta t} dt \tag{1}$$

where:

$P_A$ – demand for the winner's output, equal to its production capacity,
$P_B$ – demand for the other bidder's output, equal to its production capacity,
$\beta$ – a discount rate,
$C$ – a product unit cost,
$\vartheta(t)$ – increase in the firm's output and in the demand for it,
$\gamma(t)$ – increase in costs as a result of winning the auction,
$e$ – Euler's number.

The first element in formula (1) is the winner's return from its regular production plus the project. The second element is the return obtainable between the actual date of completing the project and the end of the maximum period stated in the bid. Elements three and four stand for operational costs in these two periods. To determine the profitability of delivering the project in the offered time $t^*$ (under the assumed values of $\vartheta(t)$ and *γ(t)*, the return that could be made by staying passive also needs to be considered. Assuming that the product

[1] W. Łyszkiewicz (2000) mentions an individual auction (otherwise an English auction), public auctions and hybrid auctions, p. 325.

the bidders are offering is homogenous, the return of the passive firm will be defined by the following formula:

$$Z_{pas} = \int_{0}^{t^*} (P_B(t) \cdot C(P_B(t) + P_A(t)) + \vartheta(t)) \cdot e^{-\beta t} dt + \int_{t^*}^{T} (P_B(t) \cdot C(P_B(t)) \cdot e^{-\beta t} dt + \\ - \int_{t^*}^{T} K_B(P_B(t)) \cdot e^{-\beta t} dt \tag{2}$$

As the production capacity of firms tendering for a project is similar in most cases, the difference between the project completion times they will offer will be insignificant. The criterion for deciding whether or not to pursue a project is the following:

$$Z_{atk} > Z_{pas} \tag{3}$$

If it is met, then it is profitable for a firm to increase its costs and to outbid its competitor(s). However, because of variations in the factors responsible for supply growth (e.g. resulting from the changing market situation) and the project execution costs that are the greater the shorter the completion time, it is difficult to establish without a computer simulation which bidder is at an advantage. Assuming that firms offer homogenous products and that an additional project will have a different impact on their costs, the more probable winner of the tender is the one that is technologically more advanced. However, if the project completion time $t^*$ a firm considers is too short, then a passive stance may be more profitable. This conclusion holds true for any number of auction participants regardless of whether their technologies are homogenous or heterogeneous. Figure 2 shows the evolution of costs with firms' output changing in time.

Curve B in variant 1 is representative of a firm that operates much below its capacity, so it can increase its output without additional costs having to be incurred (the hyperbolic dependence of costs on the proposed project execution time means that the sum of the project's costs is assumed to be constant). The curves in variant 2 shows a situation where output increase comes with additional costs, such as higher rates of overtime paid to the workers (a linear cost curve for output increase). In variant 3, both firms will have to make additional expenditures to increase the amount of capital assets or to purchase new technologies, etc. In variant 4, additional expenditures on human resources are necessary to cut back the project's duration (more workers will have to be recruited).

The optimization models (1) and (2) do not include factors related to the uncertainty of demand in time $t \in T$. Firms with a stronger market position and therefore capable of penetrating the market at lower costs in a longer period (even in $t > T$) are at an advantage, as well as firms with greater technological experience that learn by doing it in the long run. The mathematical model the authors put forward may be expanded as needed, but then the number of variables must be appropriately increased and a multi-criterion analysis must be applied to find the solution.

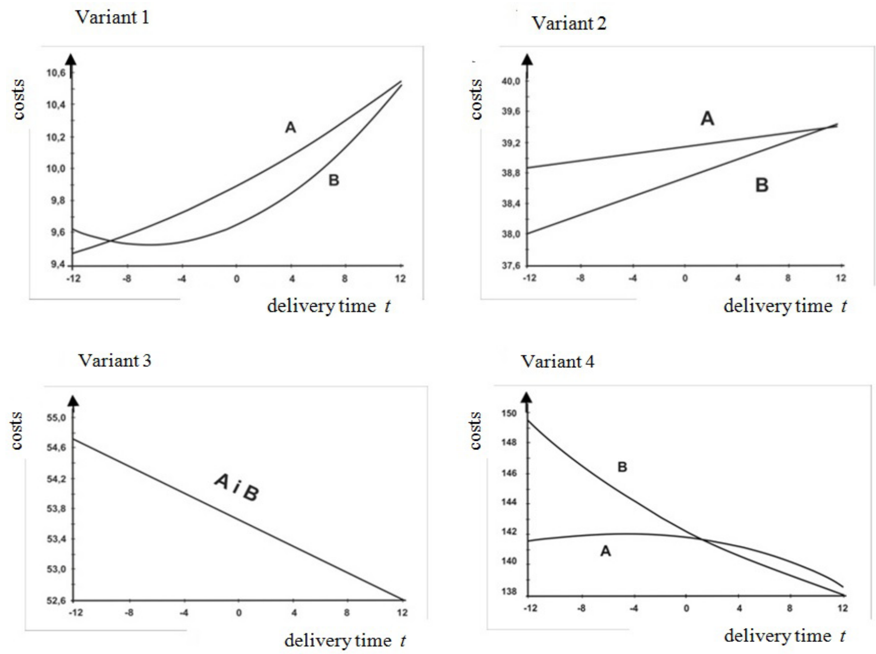


**Figure 2.** Variants of curves showing the costs of companies A and B in relation to the project execution time

Source: developed by the authors.

## Example

To make the presentation simpler, let us consider a case where only two firms, A and B, enter into a public auction (this simplification only changes the evolution of total demand for the given product and, indirectly, the level of its price). The auction involves an infrastructure project and the bids are evaluated against the proposed project completion times. Other variables (price, performance, service quality, terms of warranty, etc.) are as per tender documentation and are identical for both contractors because of the homogeneous character of the service put out to tender.

The parameters of both firms' production are estimated using the historical data presented in table 1.

The forecasting related to the value of production, unit cost and profit over the next 15 months. It should be noted that these projections do not take into account the effects of participation in the auction and only reproduce the past trends of both companies on the relevant public procurement market. Figure 3 shows diagrams which graphically show a simulation of the predicted categories.

**Table 1**

Historical data on firms A and B

| Period | Months t | Firm A | | Firm B | | Unit cost (market) (PLN millions) |
|---|---|---|---|---|---|---|
| | | output (units) | unit cost (PLN millions) | output (units) | unit cost (PLN millions) | |
| II period | –12 | 9.50 | 39.8 | 9.35 | 37.9 | 51.2 |
| | –11 | 9.55 | 38.9 | 9.35 | 38.3 | 54.3 |
| | –10 | 9.55 | 39.1 | 9.25 | 37.4 | 54.1 |
| | –9 | 9.59 | 38.8 | 9.25 | 37.9 | 53.9 |
| | –8 | 9.60 | 38.9 | 9.30 | 38.3 | 53.6 |
| | –7 | 9.65 | 39.1 | 9.25 | 38.1 | 53.4 |
| I period | –6 | 9.70 | 39.0 | 9.30 | 38.5 | 53.6 |
| | –5 | 9.71 | 39.1 | 9.30 | 38.3 | 54.0 |
| | –4 | 9.80 | 39.2 | 9.40 | 38.3 | 53.6 |
| | –3 | 9.85 | 39.1 | 9.30 | 38.5 | 53.5 |
| | –2 | 9.80 | 39.5 | 9.35 | 38.3 | 53.7 |
| | –1 | 9.90 | 39.7 | 9.40 | 38.6 | 53.9 |

Source: developed by the authors.

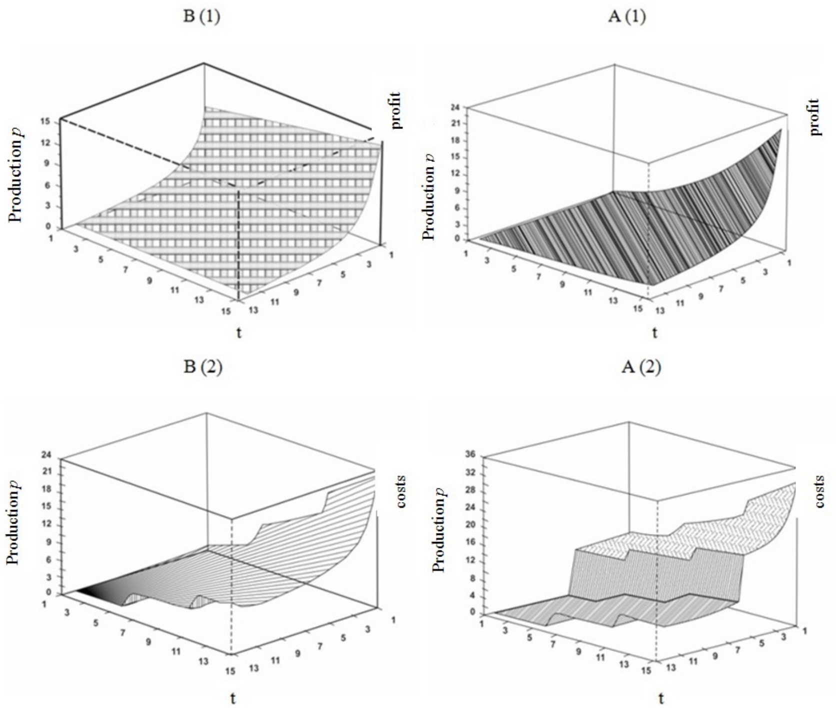


**Figure 3.** Visualisation of the estimated parameters of production, unit costs, profits for A and B

Source: developed by the authors.

The study indicates a decrease in profit, which could be associated with the decreased rates and rising costs of both companies. The study also found that higher profits from B in the earlier period of the forecast period (diagram 1) were the result of its lower production costs. However, at the end of the forecast period, both of the firm's own production (without the participation in the auction) and the costs of both companies B and A, while maintaining the current trend may be close (respectively diagram B (2) and A (2)).

Let us assume now that an auction involves the delivery of 60 units, and that the maximum delivery time is 12 months, and that both firms enter into the auction. The winner's turnover will increase by an amount equal to the value of $\vartheta(t)$, but at additional costs needed to perform the contract, which are expressed by the function $\gamma(\vartheta(t),t)$.What complicates the calculation of the likely profit from being granted the contract is the fact that the amount of the product will consequently increase in the market, so its price will probably fall[2], automatically bringing down the profits of the other firm. Let us note that if there were more bidders all of them would suffer from lower profits. The decision-making criterion for firm A is given as:

$$\begin{aligned} Z_A = \int_0^{t^*} \vartheta(t) \cdot C(P_A(t) + P_B(t) + \vartheta(t)) \cdot e^{-\beta t} dt + \\ -(\int_0^{t^*} K_A(P_A(t) + \vartheta(t)\gamma(t)) \cdot e^{-\beta t} dt - \int_0^{t^*} K_A(P_A(t)) \cdot e^{-\beta t} dt) \end{aligned} \tag{4}$$

and analogously for B

$$\begin{aligned} Z_B = \int_0^{t^*} \vartheta(t) \cdot C(P_B(t) + P_A(t) + \vartheta(t)) \cdot e^{-\beta t} dt + \\ -(\int_0^{t^*} K_B(P_B(t) + \vartheta(t)\gamma(t)) \cdot e^{-\beta t} dt - \int_0^{t^*} K_B(P_B(t)) \cdot e^{-\beta t} dt) \end{aligned} \tag{5}$$

If winning the auction only results in the costs of additional production, the above formula can be transformed into:

$$\widetilde{Z}_A = \int_0^{t^*} \vartheta(t) \cdot C(P_A(t) + P_B(t) + \vartheta(t)) \cdot e^{-\beta t} dt - \int_0^{t^*} K_A(\vartheta(t) \cdot \gamma(t)) \cdot e^{-\beta t} dt \tag{6}$$

and respectively

$$\widetilde{Z}_B = \int_0^{t^*} \vartheta(t) \cdot C(P_B(t) + P_A(t) + \vartheta(t)) \cdot e^{-\beta t} dt - \int_0^{t^*} K_B(\vartheta(t) \cdot \gamma(t)) \cdot e^{-\beta t} dt \tag{7}$$

[2] Because of the law of demand.

Taking an additional assumption that production is evenly distributed over time, we arrive at $\vartheta(t) = 60/t^*$. Another assumption is that reducing the project execution time by each of the 60 months increases production costs by 4.5% in relation to the costs of delivering the project in the maximum time stated in the tender documentation. Further, each time output grows by 100% costs increase by 5.5%.The results of the computer simulation performed with an annual discount rate of 6% are shown in Table 2.

**Table 2**

Computer simulation of profits

| Profit before and after simulation (PLN millions) | Delivery time t offered by a firm (months) | | | | | |
|---|---|---|---|---|---|---|
| | 7 | 8 | 9 | 10 | 11 | 12 |
| $Z_A$ | – | – | – | –216.6 | **73.7** | **393.0** |
| $Z_B$ | – | – | – | –194.9 | **89.5** | **402.9** |
| $\tilde{Z}_A$ | –170.1 | –9.4 | **135.9** | **270.8** | **397.8** | **518.7** |
| $\tilde{Z}_B$ | –152.9 | **5.9** | **149.7** | **283.1** | **408.6** | **528.0** |

Source: developed by the authors with the Scilab software package.

Table 2 shows that considering the character of the auction and the assumptions about the distribution of costs, firm B has a slight advantage. Even with its smaller output, it can outbid firm A because its production costs are (initially) lower. Nevertheless, the differences between the two firms are so small that even minor changes in the circumstances of the auction (mainly in market variables and costs) may reverse the situation. The same may occur when A and B have different effects of learning processes. According to the findings of this research, firm B does not benefit from learning processes[3].

## 3. Bidding strategies

Firms preparing their bids make their decisions under uncertainty (Tymiński 2003). An important fact is that the shorter the project execution time, the higher probability of winning the tender and a lower the probability of future profits.

A firm preparing a bid faces the dilemma of choosing a bid value maximising the expected return. For an optimal bidding strategy to be developed for an auction the competitors' offers must be competently estimated and the minimum expected return must be defined. This is not easy and sometimes even impossible to achieve. The problem can be solved in several ways. One of them is a computer simulation of bid profitability presented in this article, which can be conducted using historical data on the competitors and surveys enhanced by economic intelligence (Tymiński 2003).

[3] As shown by the function of unit costs in period I for the value of *P* (Table 1).

The effects of learning-by-doing achieved by firm A may considerably increase its output in the long term, so they should be deemed positive. It is so, because the following reduction in unit production costs may increase demand in real terms.

## Conclusions

At public auctions involving high-value projects such as the construction of roads it is the awarding entity that sets the requirements for the bidders to fulfil. In most cases, the main criteria for selecting the winner are the price, the length of time needed to perform the contract, or both. In the simplified case presented above both firms use similar production technologies, but their production parameters are different and thereby so are the production costs and production capacities. Consequently, their involvement in an additional project has different impacts on their economic effectiveness and profits. The difference in the economic effectiveness between the successful bidder and the firm that have chosen to stay passive is expressed by the delivery time *T* expected by the awarding entity and by the rate of increase in the demand for both firms' products in that time. The firm that estimates its capabilities as inferior in the given market circumstances should rather stay passive[4]. The numerical procedure underlying the preparation of this article has been developed by the authors themselves from the Scilab software package. The system is flexible in handling many different variants of the distribution of project costs and market variables. The example presented in the article involves two firms, but in fact any number of firms can be analysed, as well as organisations offering different products and services.

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[4] Learning processes are important in this case. Firm A utilizes the processes so it has a greater potential for meeting demand.

**OPTYMALIZACJA OFERTY W AUKCJI PUBLICZNEJ DOTYCZĄCEJ INWESTYCJI INFRASTRUKTURALNEJ PRZY WYKORZYSTANIU PAKIETU PROCEDUR NUMERYCZNYCH**

**Streszczenie:** W artykule poruszono tematykę związaną z finansową stroną uczestnictwa w aukcjach publicznych inwestycji infrastrukturalnych. Omówiono uwarunkowania związane z przystąpieniem do przetargu. Szczególną uwagę autorzy zwrócili na analizę kosztów stanowiących ważny element w procesie przetargowym. Artykuł zawiera przykład zastosowania narzędzi informatycznych, opracowanych w oparciu o platformę numeryczną Scilab. Symulacje finansowe dotyczą dwóch firm rywalizujących o realizację inwestycji infrastruktury drogowej uruchomionej poprzez aukcję publiczną co do czasu realizacji inwestycji, a przykład ilustruje możliwości analizy opłacalności inwestycji z uwzględnieniem warunków rynkowych, tj. zmieniających się kosztów inwestycji i stopy dyskontowej.

**Słowa kluczowe:** inwestycje, przetarg, optymalizacja ofert, procedury numeryczne, program Scilab

## Cytowanie

Juszczyk S., Tymiński J. (2014), *Optimisation of Bids for a Public Auction Involving the Delivery of an Infrastructure Project Using a Package of Numerical Procedures*, Zeszyty Naukowe Uniwersytetu Szczecińskiego nr 802, „Finanse, Rynki Finansowe, Ubezpieczenia” nr 65, Wydawnictwo Naukowe Uniwersytetu Szczecińskiego, Szczecin, s. 1–11; www.wneiz.pl/frfu.